A Robust and Efficient First Order Reliability Method for Structural Reliability Analysis

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1. Introduction

The Hasofer-Lind and Rakwitz-Fiessler (HL-RF) algorithm [1] is a classical and widely used reliability analysis algorithm for structures in the first-order reliability method (FORM). However, it may produce the non-convergence phenomenon of periodic oscillation when dealing with highly nonlinear functions [2]. The improved algorithms, such as stability transformation method (STM) [3] and relaxed HL-RF (RHL-RF) [4] algorithm, can deal with these problems, but they are inefficient. That is to say, although these algorithms have made some progress in various aspects, there are still some problems such as low robustness and poor efficiency in practical engineering. Therefore, it is very important to research a robust and efficient reliability analysis algorithm for the application and popularization of reliability analysis methods in engineering.

In this paper, an adaptive finite step length (AFSL) algorithm is proposed for structural reliability analysis based on the FSL algorithm [5] and Armijo line search method [6]. The sensitivity factor with a step length is implemented to improve the robustness, and an adaptive optimization method based on Armijo line search is proposed to improve the efficiency. In addition, the initial step length and adjusting coefficient are optimized to make the algorithm more suitable for various nonlinear functions.

2. Proposed Algorithm

Firstly, the random variables $\boldsymbol{X}$ should be transformed into standard normal random variables $\boldsymbol{U}$. The transformation is expressed as [7]

$$\boldsymbol{U} = \Phi^{-1}\left[F_{\boldsymbol{X}}\left(\boldsymbol{X}\right)\right] \tag{1}$$

where $F_{\boldsymbol{X}}$ is the cumulative distribution function (CDF) of $\boldsymbol{X}$ and Φ^{-1} is the inverse CDF of the standard normal distribution. The LSF in standard normal distribution space is converted to $G\left(\boldsymbol{U}\right)$.

Then the most probable point (MPP) $\boldsymbol{U}^*$ and the reliability index $\beta = \left\|\boldsymbol{U}^*\right\|$ can be searched by following iteration formula:

$$\boldsymbol{U}^{k+1} = \frac{\nabla^{\mathrm{T}} G\left(\boldsymbol{U}^k\right)\boldsymbol{U}^k - G\left(\boldsymbol{U}^k\right)}{\nabla^{\mathrm{T}} G\left(\boldsymbol{U}^k\right)\boldsymbol{\alpha}^k}\boldsymbol{\alpha}^k \tag{2}$$

where $\nabla G\left(\boldsymbol{U}^k\right)$ is the gradient of $G\left(\boldsymbol{U}^k\right)$ and the sensitivity factor $\boldsymbol{\alpha}^k$ is calculated by

$$\boldsymbol{\alpha}^k = \frac{\boldsymbol{U}^k - \lambda^k \nabla G\left(\boldsymbol{U}^k\right)}{\left\|\boldsymbol{U}^k - \lambda^k \nabla G\left(\boldsymbol{U}^k\right)\right\|} \tag{3}$$

where λ^k is the step length.

If $\left\|\boldsymbol{U}^{k+1} - \boldsymbol{U}^k\right\| \geq \left\|\boldsymbol{U}^k - \boldsymbol{U}^{k-1}\right\|$, set $\lambda^{k+1} = c\lambda^k$, where $c \in \left[0.5, 0.6\right]$ is the adjusting coefficient, and the initial step length is defined as $\lambda^0 = 50$. In addition, the iteration point is optimized by $\boldsymbol{U}_*^{k+1} = \boldsymbol{U}^k + \theta^k\left(\boldsymbol{U}^{k+1} - \boldsymbol{U}^k\right)$, in which the optimized coefficient θ^k can be calculated by

$$\theta^k = \max_b\left\{\omega^b \left| G\left(\boldsymbol{U}^k + \omega^b \boldsymbol{d}^k\right) - G\left(\boldsymbol{U}^k\right) < 0\right.\right\} \tag{4}$$

where $\omega = 0.5$.

Finally, the failure probability is estimated by

$$P_f \approx \Phi\left(-\beta\right) \tag{5}$$

where Φ is the CDF of the standard normal distribution.

3. Illustrative example

A highly nonlinear function is given as [8]

$$G_1 = X_1^4 + 2X_2^4 - 20 \tag{6}$$

where X_1 and X_2 are normal random variables with $\mu_1 = \mu_2 = 10$ and $\sigma_1 = \sigma_2 = 5$.

The convergent results including the number of iterations, G evaluations and reliability index for all algorithms are listed in Table 1. As seen, all the STM, RHL-RF, FSL and AFSL algorithms converge to stable solutions, but the HL-RF algorithm fail to converge because its iterative process falls into a periodic loop. The reliability index calculated by MCS with 10^6 samples is 2.8998. The AFSL algorithm converge to the stable result as $\beta = 2.3842$, which is more accuracy than the other algorithms. In addition, it is obvious that the proposed AFSL algorithm is the most efficient among these algorithms, according to the number of iterations and G evaluations.

Table 1. Convergent results

Algorithms	Iterations	G-evaluations	β
HL-RF	-	-	-
STM	154	308	2.3654
RHL-RF	181	542	2.3654
FSL	108	216	2.3655
AFSL	26	73	2.3842

4. Conclusion

In this paper, a robust and efficient algorithm for structural reliability analysis is presented. The development of AFSL algorithm is motivated by the non-convergence phenomenon of HL-RF algorithm and the inefficiency of several existing FORM algorithms. Firstly, a step length parameter is introduced into the sensitivity factor of HL-RF algorithm. It can avoid the iterative process falling into periodic loop, making the proposed algorithm more robust than HL-RF algorithm. Secondly, an adaptive iteration point optimization method is proposed based on the sufficient descent condition with Armijo line search. It can optimize the iteration points with large errors in the iterative process, making the proposed algorithm more efficient than several existing FORM algorithms. Finally, the initial step length and adjusting coefficient are optimized, making the proposed algorithm more suitable for various nonlinear functions. The robustness and efficiency of the proposed AFSL algorithm are compared with several FORM algorithms, such as the HL-RF, STM, RHL-RF and FSL algorithms. From the example, it can be concluded that the proposed algorithm is not only more robust than HL-RF algorithm, but also more efficient than the STM, RHL-RF and FSL algorithms. Consequently, the proposed AFSL reliability analysis algorithm is very useful for engineering application due to its good performance with high robustness and efficiency.

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