A Response-adaptive Method for Design of Validation Experiment

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1. Introduction

As engineering decisions have increasingly relied on the predictive results of computational models instead of repeated product prototyping and testing, the fidelity of computational models has become important. In recent years, model verification and validation (V&V) methodology that improves and assesses predictive capability of computational models [1-3] has been studied to build computational models with high predictive capability.

For the model validation, it is desirable to build a set of experimental responses extracted from a sufficient number of test samples; ideally, infinite number of test samples. Then, the agreement between computational and experimental responses can be evaluated without any uncertainty. However, in reality, resources for testing (e.g., time, budget, facilities) are limited.

To address this issue, this paper proposes a new methodology of designing validation experiments named as the response-adaptive experiment design (RAED), to reduce type II error. The RAED helps to effectively allocate validation resources (samples) within the validation domain and to increase possibility of rejecting an invalid computational model when the number of experimental data is limited.

2. Response-adaptive Experiment Design

The response-adaptive experiment design (RAED) is presented in this section. The RAED helps an engineer decide a better experimental condition among many candidates as the next experiment. Obtaining test datum at the selected condition helps to reduce type II error without increasing type I error (or significant level) rather than taking data at other experimental conditions. The flow diagram of the RAED is shown in Fig. 1.

The relation between low area metric and reduction of type II error is demonstrated by comparing type II errors of two experimental conditions below.

Experimental condition I: The true distribution of experimental results is standard normal distribution (f0 ~ Normal(0,1)), and the PDF of predicted results is normal distribution (f1 ~ Normal(0.56, 1)). Eighteen experimental data are randomly obtained from the f0 to calculate area metric.

Experimental condition II: The true distribution of experimental results is also standard normal distribution (f0); however, the PDF of predicted results is normal distribution (f2 ~ Normal(1.05,1)). Eighteen experimental data are randomly obtained from the f0 to calculate area metric.

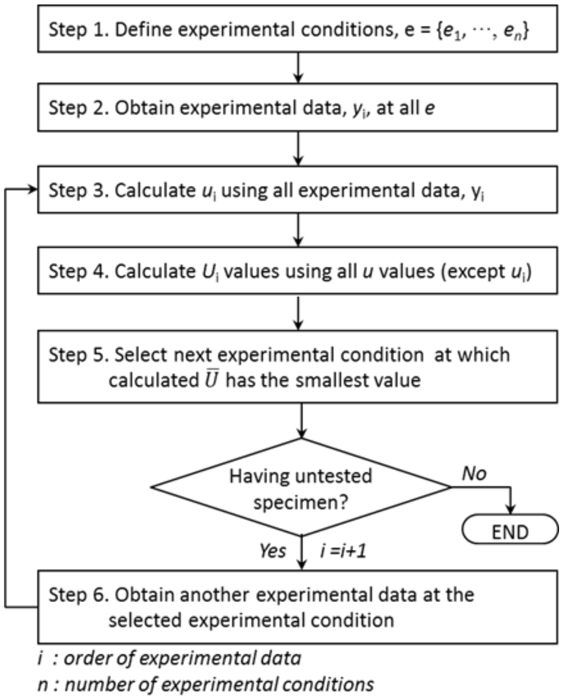

Fig. 1 Flow diagram of response-adaptive experiment design

It is definite that the predicted model, f_2, is more invalid and has larger area metric than the predicted model, f_1, when they are compared with the true distribution of experimental results, f_0. Type II error of each experimental condition can be calculated by using the PDF of area metric by following equation.

$$\text{Type II error } = \int_0^{D_t(\alpha)} f_{u,i}(x)dx \qquad (1)$$

For the quantification, it is assumed that type I error is 0.1. The procedure of constructing the PDFs of area metric is described as follows:

- Step 1: Determine a virtual sampling size (k) and experimental data size (i). Here, k=5,000 and i=18.
- Step 2: Generate samples (y_1, y_2, ⋯,y_i) randomly from the true distribution of

experimental data.

- Step 3: Calculate the CDF values (u_1, u_2, ⋯, u_i) corresponding to y_i.
- Step 4: Calculate the area metric, U_m.
- Step 5: Repeat Step 2-4 k times to generate random data (U_1, U_2, ⋯, U_k) and then construct empirical PDF of area metric using U values. In this paper, the pearson system is used to construct empirical PDFs of U values.

The difficulty in designing experiments for model validation is that we do not have any idea on the true distribution of the experimental data. Thus, the RAED has to employ an adaptive process to determine next experimental condition based on pre-obtained experimental results.

3. Conclusions and Future Work

This paper presents a new method for designing validation experiments named as the response-adaptive experiment design (RAED). Up to now, the hypothesis test for validity check only considers type I error (or significance level) to evaluate whether a computational model is valid based on given experimental data. It is because quantification of type II error is not feasible without knowing the true distribution of experimental data. While proceeding model validation activities, the RAED helps to effectively allocate validation resources (samples) at experimental operating conditions and to increase possibility of rejecting an invalid computational model when a number of experimental data is limited.

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