A rate-dependent failure criterion based on distortion strain energy density

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1. Abstract

A strain-rate dependent failure criterion is proposed for orthotropic materials based on rate-dependent constitutive model and the concept of distortion strain energy density. In a linear elastic orthotropic constitutive model, stresses are decoupled into two parts, static stress and dynamic stress in which a strain rate is introduced in a form of power function. By subtracting volumetric strain energy density from total strain energy density, the so-called dynamic distortion strain energy density is then derived rigidly to establish strain-rate dependent failure criterion. According to published test data, parameters in the failure criterion established are fitted, and dynamic failure strain and stress are depicted versus strain rate and parameters. Some common sense about strain rate effect is explained quantitatively.

2. Introduction

Under various loading speeds, some materials such as fiber reinforced composites exhibit different strengths. From the view of continuum mechanics, seldom failure criterion is able to predict such dynamic strength reasonably and effectively. So, this paper attempts to construct a function involving strain rate as a failure criterion based on distortion strain energy density.

3. Strain-rate-dependent constitution

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} + \mathbf{F}\dot{\boldsymbol{\varepsilon}}^{n} \tag{1}$$

Constitutive equation considering strain rate effects is composed of two parts as shown in Eq.(1), where $\mathbf{C}$ is the elastic stiffness and $\mathbf{F}$ is the coefficient matrix representing the influence of each strain rate component.

The dilatational strain energy density υ_{V} can be derived as:

$$\upsilon_{V} = \frac{1}{2}\left(\sum_{i,j=1}^{3} k_{ij}\varepsilon_{m}^{2} + \sum_{i,j=1}^{3} f_{ij}\dot{\varepsilon}_{m}^{n}\varepsilon_{m}\right) \tag{2}$$

The total strain energy density υ_{ε} under dynamic loading is derived as:

$$\upsilon_{\varepsilon} = \frac{1}{2}\sum_{i=1}^{6}\sigma_{i}\varepsilon_{i} = \frac{1}{2}\left(\sum_{i,j=1}^{6} k_{ij}\varepsilon_{i}\varepsilon_{j} + \sum_{i,j=1}^{6} f_{ij}\varepsilon_{i}\dot{\varepsilon}_{j}^{n}\right) \tag{3}$$

On the basis of the reference of the concept of distortional strain energy density, the distortional strain energy density υ_{d} is calculated by subtracting υ_{V} from υ_{ε}:

$$\upsilon_{d} = \upsilon_{\varepsilon} - \upsilon_{V} \tag{4}$$

4. Examples

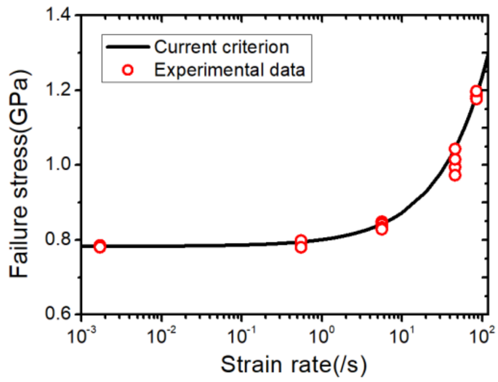

Fig.1 Comparison of calculations and experiments of longitudinal tension.

Figure shows the failure stress of composites laminates obtained by the current criterion and experiment. Power law exponent n determines the rate-dependency of the material, and it gives a better match with experimental data that n equals 0.7.

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References

[1]. Yehia N A B. Distortional strain energy density criterion: the Y-Criterion. Engineering Fracture Mechanics. 1991; 39(3):477-485.
[2]. Yang W H. A Generalized von mises criterion for yield and fracture. Journal of Applied Mechanics. 1980; 47(2):297-300.