Vulnerability assessment procedure of naval ship according to a shot lines on M&S

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1. Introduction

The Survivability of naval ship can be defined as its capability to avoid or withstand a man-made hostile environment. The survivability of a naval ship consists of three probabilities: susceptibility, vulnerability, and recoverability. Especially the vulnerability is the probability to withstand the man-made hostile environment. As the first step to assess the vulnerability, it is necessary to define the shot line on the target area. In order to define the hit location, the model of probability distribution of the shot line reaching the target must be considered together with the motion equation of the warhead. In this paper we present a method for defining random shot line using a multivariate distribution model about the distribution of hit location. Also considering the warhead motion, this paper shows a procedure of vulnerability assessment for penetration effect based on the shot line and damage function.

2. Weapon Effectiveness

Fig. 1 is an example of weapon system according to threat characteristics and effectiveness. In this study we assume the combat situation as aircraft vs warship with single penetration threat.

The equation of warhead motion should be analyzed by using parameters such as initial launch velocity, altitude, range and resistance against flight of the projectile [1]. Table 1 shows the equations to calculate the impact parameters to analyze penetration. And, it is assumed that there is no drag force that is received as the warhead moves.

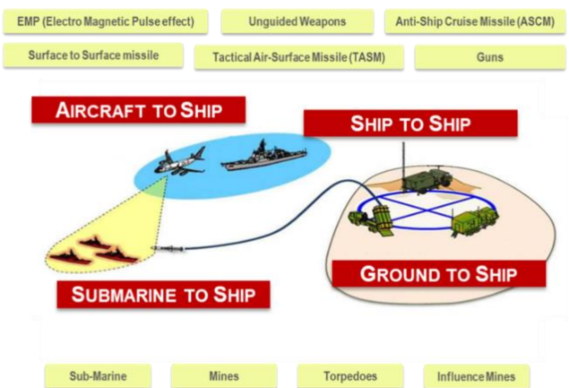

Fig. 1 Threat on combat environment

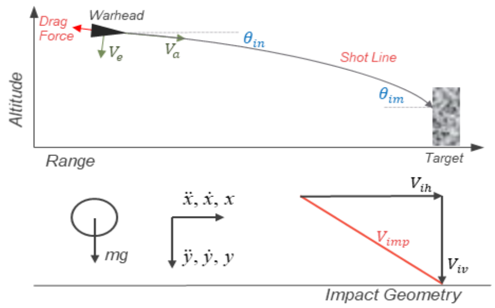

Fig. 2 Warhead motion and impact geometry

Table 1 The equations of warhead parameters

Horizontal initial velocity	$V_h = V_a \cos\theta_{in} - V_e \sin\theta_{in}$
Vertical initial velocity	$V_v = V_a \sin\theta_{in} + V_e \cos\theta_{in}$
Location (by time)	$\begin{cases} x = V_{oh} \cdot t \\ y = V_{ov} \cdot t + \frac{1}{2} g t^2 \end{cases}$
Impact Angle	$\theta_{im} = \tan^{-1}\left[\frac{V_{iv}}{V_{ih}}\right]$
Impact Velocity	$V_{imp} = \sqrt{V_{ih}^2 + V_{iv}^2}$
Time of Flight	$TOF = \frac{-V_v + \sqrt{V_v^2 + 2gh}}{g}$

3. Shot Line and hit location

The hit location where the warhead reaches the target is related to the start of survivability assessment procedure and is dispersed in a form including the miss distance [1]. In this study we define a random shot line and hit location as coordinates in 3D space using multivariate distribution model based on standard normal distribution. Fig. 3 shows the hit location and coordinates for a single penetration hit. To show this location, we use a multivariate distribution model combined with standard normal distribution (Eq. 1).

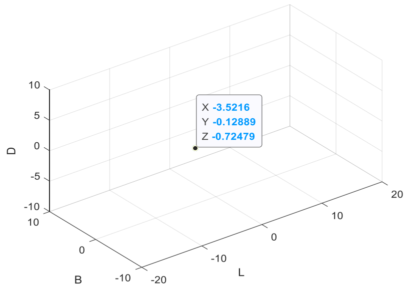

Fig. 3 Random hit location for single penetration hit

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0_1 & \sigma_2^2 & \ldots & 0 \\ \vdots & \ldots & \ldots & \vdots \\ 0 & \ldots & \ldots & \sigma_n^2 \end{pmatrix} = Z \sim N_n(0_n, 1_n) \tag{1}$$

4. Vulnerability Assessment Procedure

In this study we define the vulnerability assessment procedure of penetration effect as 3 steps [2]. Firstly, analyze the penetration effect according to the coordinates from the shot line. At this step, penetration analysis is performed repeatedly on the direction of the warhead, when the warhead speed reaches to zero or until can't penetration on the target (Eq. 2).

Secondly, define the critical components, and analyze which components will be damaged for the penetration effect. At this step, the area of critical components should be defined as simplified method Such as AABB (Axis Aligned Bounding Box) [3].

Finally, define the damage model, and calculate the probability of damage based on damage functions such as Cookie-Cutter or Carleton damage function. After defining the probability of damage, the vulnerability can be evaluated according to the component's redundancy and layout. The calculation of the vulnerability is based on the ratio of the presented area to its vulnerable area (Eq. 3), and the area can be described as the multiplication of the probability of damage and presented area of critical components (Eq. 4).

$$V_{50} = \sqrt{\frac{2 \cdot L \cdot G_d \cdot t^2}{m \cdot \cos^2\theta}}, \quad V_{residual} = \sqrt{\frac{V_i^2 + V_{50}^2}{1 + \frac{\rho \cdot A_p \cdot t}{W \cdot \cos\theta}}} \tag{2}$$

$$P_K = A_V / A_p \tag{3}$$

$$A_v = P_d \cdot A_p \tag{4}$$

The probability of damage that a threat weapon can damage the aimed target can be provided from damage functions [4]. Eq. (5) shows the Carleton damage function, where WR_r and WR_d are defined as the weapon radius to the range and deflection directions. This means the expected value that warhead will deviate from aim point. In one-dimensional model with Gaussian distribution, the probability of damage for single shot can be defined as Eq. 6 and 7 for each direction (Fig. 21).

$$P(x,y) = \exp\left[-\frac{4x^2}{(2 \cdot WR_r)^2} - \frac{4y^2}{(2 \cdot WR_d)} \right] \tag{5}$$

$$PD_x = g(x) \cdot c(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2\sigma_u^2} \right] dx \tag{6}$$

$$PD_y = g(y) \cdot c(y) = \frac{1}{\sigma_y \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{y^2}{2\sigma_u^2} \right] dy \tag{7}$$

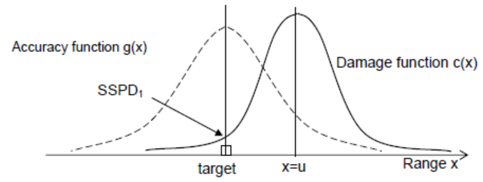

Fig. 4 single warhead on unitary target

Conclusion

In this paper, we propose a theoretical procedural model to assess the vulnerability of naval ship. And we use the multivariate distribution model to specify the coordinates of the 3D shot location. In case of vulnerability, the analysis model of critical components was defined for the threat of a single penetration shot, and the vulnerable area of critical components was calculated by the probability of damage based on damage function. This theoretical model based on shot line and damage function will be expanded and modified in the future for threats of multiple penetration and explosion effects.

References

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