

Simplified Vibration PSD Synthesis Method for MIL-STD-810

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1. Introduction

MIL-STD-810, the U.S. Army's environmental test specification, details the synthesis method of various vibration conditions using Fatigue Damage Spectrum(FDS) since version G w/Change 1 released in 2014[1]. FDS is the cumulative damage per frequency of virtual components using the strength of the input vibration and associated exposure time. It is used to compare the severity of different vibration test specifications or to synthesize vibration stress from various events. Although it is more accurate to derive FDS directly from mode-specific time waveforms, the MIL-STD-810 uses formula to obtain FDS directly from the input PSD, which makes it easy to convert PSD and FDS back and forth. Derivation of this formula assumes that the probability distribution of the maximum values of the response waveform follows a known function. This assumption may result in inaccurate cumulative fatigue damage, for example, when the time waveform is transient rather than random. In this paper, the formula for deriving the FDS from the PSD in MIL-STD-810 is theoretically reviewed and then data measured in real-world field such as automobiles and excavators were synthesized using this formula and the results were compared with the results synthesized using commercial software to check the validity and limitations. Furthermore, the modified and simplified formula is proposed to combine PSDs of vibration events and associated exposure times into a test vibration PSD and associated run time. In areas where vibration characteristics are well known as random, it is expected that if vibration conditions are synthesized using the formula proposed in this paper, the procedure will be intuitive and simplified to increase work efficiency.

2. FDS (Stress histogram)

The calculation of FDS is based on the response of Single Degree of Freedom (SDOF) system, which is so called the virtual specimen, to the input signal. It has been shown that the pseudo-velocity response is more closely related to stress than other response measures [2]. The relationship between pseudo velocity and stress is roughly proportional. Once the stress time history is given, stress histogram can be derived by counting method and damage index can be calculated for

each stress level in case S-N curve is given (in practice the coefficients are assumed) and then the total damage can be simply summed by the cumulative damage law. The FDS of the frequency range of interest is derived by changing the resonant frequency of the virtual specimen and repeating the entire process. Fig. 1 shows the procedure.

3. FDS (Rayleigh distribution)

The previous section described the process of obtaining response signals for input signals and obtaining cumulative damage by counting stress. Now, In SDOF systems with low damping ratios, we use the fact that the peak probability density function of the response is known to be approximately a Rayleigh distribution, which means that we do not need a stress histogram to calculate FDS [3]. Because the relationship between pseudo velocity and stress is roughly proportional, it follows that the fatigue damage in the virtual system can be integrated if S-N curve is given. The cumulative damage for each natural frequency of SDOF is given in Eq. (1) [4].

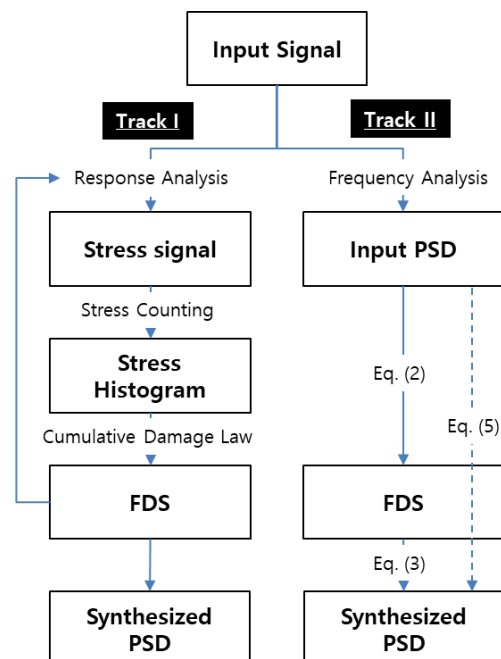


Fig.1 Procedures for deriving FDS and PSD from input data. The left track is a procedure based on the original concept of FDS and the right one is based on MIL-STD-810 procedure

$$D(f_n) = \frac{f_n T}{c} \left[\frac{k^2 G(f_n)}{8\pi f_n \zeta} \right]^{b/2} \Gamma[1 + b/2] \quad (1)$$

where;

f_n = natural frequency of test item
 T = total duration of exposure to the dynamic environment
 b, c = material constants
 k = constant of proportionality
 $G(f_n)$ = PSD of input acceleration at the test item resonance frequency
 ζ = damping ratio of test item
 $\Gamma[\]$ = Gamma function of $\lbrack \rbrack$

The Eq. (2) below is named as the "Damage potential spectrum" after the Eq. (1) has been sorted out by excluding the constant terms. The original usage of Equation (2) is a descriptor to compare the severity of various test conditions [4]. But, in MIL-STD-810, the cumulative fatigue damage caused by various vibration conditions is obtained using Eq. (2) and linearly combined under the cumulative law.

$$DP(f_n) = f_n T \left[\frac{G(f_n)}{f_n \zeta} \right]^{b/2} \quad (2)$$

Furthermore, the equation of deriving G from the DP itself is presented like Eq. (3), which is directly drawn from Eq. (2)

$$G(f_n) = f_n \zeta \left[\frac{DP(f_n)}{f_n T} \right]^{2/b} \quad (3)$$

The equations above can be accepted if the maxima probability density function is a Rayleigh distribution, which Lalanne said would be approximately acceptable only if the irregularity factor is greater than 0.6 [5].

4. Simplified PSD synthesis method

If the various vibration conditions are defined as $G_i(f_n)$ s and T_i s, damage potential DP_i for each condition can be obtained using Eq. (2). Since the cumulative damage law can be applied to the obtained DPs to have DP_{total} . Now, DP_{total} and the test time, T_{test} , is inserted in Eq. (3) to have Eq. (5) through the process of Eq. (4).

$$\begin{aligned} G_{test}(f_n) &= f_n \zeta \left[\frac{DP_{total}(f_n)}{f_n T_{test}} \right]^{2/b} \\ &= f_n \zeta \left[\sum_i \frac{T_i}{T_{test}} \left(\frac{G_i(f_n)}{f_n \zeta} \right)^{b/2} \right]^{2/b} \end{aligned} \quad (4)$$

$$\therefore G_{test}(f_n) = \frac{1}{T_{test}^{2/b}} \left[\sum_i T_i (G_i(f_n))^{b/2} \right]^{2/b} \quad (5)$$

The calculation of G_{test} is reduced to the function of input G_i s, T_i s, test time, and b , with the natural frequency and damping ratio canceled in Eq. (6)

5. Comparison of PSD synthesis methods

Based on the actual measured data, PSD synthesis was performed and the validity of the results in Section 4 was examined. The data used in the comparison are data showing random characteristics measured in the car and transient characteristics measured in the excavator. For each of the two data, the commercial software and the simplified PSD synthesis method were used to obtain and compare the results. Based on these comparison results, we discuss the possibility and limitations of the simplified PSD synthesis method.

6. Conclusion

The theoretical background of the Damage Potential of MIL-STD-810 was examined and the simplified PSD synthesis equation was derived. This equation can be used to intuitively and quickly derive synthetic PSDs, but there are areas that can not be used due to assumptions in the derivation of DP. Therefore, it is necessary to determine the characteristics of the measured vibration and to determine whether to use the formula (Section 4) or the conceptual method (Section 2).

References

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