Reliability Optimization Design of Crank-Slider Mechanism Considering Uncertainties of Distribution Parameters

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1. Introduction

For structural systems where the input variables are uncertain, even if the distribution type is known, sparse or inaccurate data will cause the distribution parameters to be unavailable. If a probability distribution is used in the distribution parameters, then family-distributed relatives can be used to represent these random variables. Therefore, for the structural systems with both the uncertainties of input variables and their distribution parameters, three sensitivity indices are proposed by Wang [1-2] based on the Sankararaman's method [3] to measure the influence of input variables, distribution parameters and their interactive effects. With those sensitivity indices, analysts can make a decision that whether it is worth to accumulate data of one distribution parameter to reduce its uncertainty. However, the effectiveness of these uncertain distribution parameters on the reliability optimization results have not been researched. Therefore, taking crank-slider mechanism as example, a reliability optimization design method considering uncertainties of distribution parameters is proposed.

2. Parametric representation of uncertain distribution parameters

Assuming the insufficient data of $\mathbf{Y}$ contains m point data $a_i\left(i=1,2,\cdots,m\right)$ and n intervals data $\left[\underline{b}_j,\bar{b}_j\right]\left(j=1,2,\cdots,n\right)$. Let $f_y\left(y|\xi,\theta_k\right)$ denote the probability density function (PDF) under candidate distribution type $\theta_k\left(k=1,2,\cdots,7\right)$ and distribution parameters ξ. Firstly, for every candidate distribution type θ_k, the likelihood estimation function $L\left(\xi,\theta_k\right)$ using the prescribed point data and interval data is constructed.

$$L\left(\xi,\theta_k\right)\sim\left[\prod_{i=1}^{m}f_y\left(y=a_i|\xi,\theta_k\right)\right]\left[\prod_{j=1}^{n}\int_{\underline{b}_j}^{\bar{b}_j}f_y\left(y|\xi,\theta_k\right)\mathrm{d}y\right] \quad (1)$$

The maximum likelihood estimations of ξ under distribution type θ_k are acquired by maximizing $L\left(\xi,\theta_k\right)$. Further, the uncertainty of distribution parameters ξ is calculated using Bayes' theorem. The probability density function $f_\xi\left(\xi|\theta_k\right)$ of the distribution parameters ξ is expressed as

$$f_\xi\left(\xi|\theta_k\right)=\frac{L\left(\xi,\theta_k\right)}{\int L\left(\xi,\theta_k\right)\mathrm{d}\xi} \quad (2)$$

3. Sensitivity analysis of uncertain distribution parameters

According to the law of total variance, the input variable main effect sensitivity index considering the distribution parameters can be equivalently described as [1]:

$$S_{xi}=E_\theta(V(Y)-E_{X_i}(V_{X_{-i}}(Y\mid X_i))) \quad (3)$$

For $E_{X_i}(V_{X_{-i}}(Y\mid X_i))$, the following equivalent equation can be derived with the assumption that the input variables are mutually independent,

$$E_{\mathbf{X}}(V_{\mathbf{X}_{-i}}(Y\mid X_i)) = \int_{\mathbf{X}}\left(\int_{\mathbf{X}_{-i}}\left(g\left(x_{-i},x_i\right)-\int_{\mathbf{X}_{-i}}g\left(x_{-i},x_i\right)f_{\mathbf{X}-i}(\mathbf{x}_{-i})\mathrm{d}\mathbf{x}_{-i}\right)^2\right)f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x} \quad (4)$$

Thus, the main effect can be equivalently expressed by the following equation,

$$S_{X_i}=E_\theta(V(Y)-E_{X_i}(g(x)-E_{X_{-i}}(Y\mid X_i))^2) \quad (5)$$

To further simplify the double-loop process, the multiplication dimension reduction of the response function $g(\mathbf{x})$ proposed by introducing logarithmic exponential transformation by Zhang [4] ,and Yun [5] uses Gaussian integral formula to process one-dimensional integral to calculate $E_{X_{-i}}(Y\mid X_i)$

$$E_{\mathbf{X}_{-i}}(Y\mid X_i)\approx[g(\mathbf{c})]^{1-n}\cdot g(c_1,...,c_{i-1},x_i,c_{i+1},..,c_n) \times\sum_{p=1}^{N'}\omega_p g(c_1,...,c_{j-1},x_j{}^p,c_{j+1},..,c_n) \quad (6)$$

where c_i is the average of the input variables and $\mathbf{c}=\left[c_1,c_2,\cdots,c_n\right]$ is the reference point of the model input variables **X**, ω_p is weight

$$\omega_{\mathrm{p}}=f_{X_j}(x_j)\cdot\Delta x \quad (7)$$

In this paper, a new method is proposed to deal with the Eq. (6) to improve the accuracy of $E_{X_{-i}}(Y\mid X_i)$. The integral global variable $X_i(i=1,2,\cdots,n)$ is equally probabilistically divided into q subintervals that do not overlap each other and fill the entire value area. The interval is determined by 3σ rules, and then equally divided into $\left(2n+1\right)$ cells, the step size h and the endpoint value between each cell are known, and then the adaptive sigma point is obtained by using the unscented transformation method in each subinterval and corresponding weights. The adaptive sigma point is used instead of the variable X_i, so that the solution obtained by the unscented transformation method in the local region is more accurate.

The specific process of obtaining sigma points and corresponding weights by using the unscented transformation method:

- (1) Generating sigma points and corresponding weights according to the original input variable probability distribution characteristics;
- (2) Nonlinear transformation of sigma points;
- (3) Approximate estimation of the moment of the output variable according to the converted sigma point and the corresponding weight.

Generally, the standard unscented transformation method is adopted, and $(2n+1)$ sigma points are taken in each subinterval, and $W_0 = 0$. Eq. (6) can be approximated as,

$$E_{\mathbf{X}_{\sim i}}(Y \mid X_i) \approx [g(\mathbf{c})]^{1-n} \cdot g(c_1,...,c_{i-1},s_j^{\,i},c_{i+1},..,c_n) \times \sum_{q=1}^{N'} \sum_{i=1}^{2n+1} g(c_1,...,c_{j-1},s_j^{\,i},c_{j+1},..,c_n) \cdot f_{s_j}(s_j) \cdot h_{s_j} \tag{8}$$

where $h_{s_j} = \frac{1}{2n+1} h$, h is the step size of the subinterval.

And let

$$E_{s_j} = \sum_{q=1}^{N'} \sum_{i=1}^{2n+1} g(c_1,...,c_{j-1},s_j^{\,i},c_{j+1},..,c_n) \cdot f_{s_j}(s_j) \cdot h_{s_j} \tag{9}$$

4. Reliability optimization algorithm

The example of a crank-slider mechanism in Ref. [6] is used for demonstration the effectiveness and application of the proposed method. The mechanism is shown in Fig. 1. The length of the crank x_1, the length of the coupler x_2 and the external force x_3 are strong statistical variables. Different from Ref. [6], the distribution parameters of material Young's modulus y_1 and the yield strength y_2 of the coupler are uncertain due to sparse sampling data. The friction coefficient z_1 and the offset z_2 are interval variables. The internal diameter d_1 and external diameter d_2 of coupler are 25mm and 60mm, respectively.

The system performance function is defined by the difference between the critical load and the axial load, which is written in Eq. (10),

$$G = G(\mathbf{X},\mathbf{Y},\mathbf{Z}) = \frac{\pi^3 y_1 \left(d_2^4 - d_1^4\right)}{64 x_2^2} - \frac{x_3\left(x_2 - x_1\right)}{\sqrt{\left(x_2 - x_1\right)^2 - z_2^2} - z_1 z_2} \tag{10}$$

Through comparing the AIC values of the seven candidate distribution types, the sensitivity between distribution parameters of $\mathbf{y}$ and performance function G are calculated, and the structural parameters are optimized based on the proposed reliability optimization method.

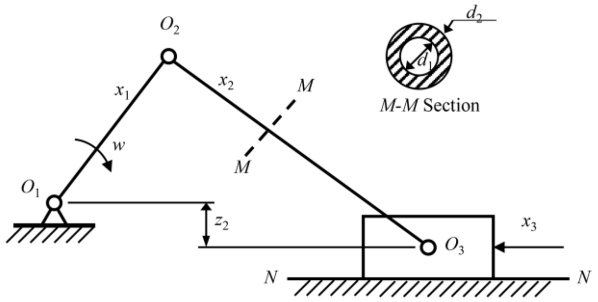

Fig. 1 A crank-slider mechanism

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