Angle cracks in functionally graded piezoelectric materials with arbitrary properties

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1. Introduction

Piezoelectric materials play a critical role in emerging technologies, particularly in the aerospace, electronic, biological and other fields. To meet the demand of advanced piezoelectric materials in lifetime and reliability, the concept of functionally graded materials has been extended to piezoelectric materials [1]. The kind of material is known as functionally graded piezoelectric materials (FGPM). Existing experiments have shown that cracks occur in FGPM because the processes of FGPM manufacture, processing, machining, and forming may introduce cracks in FGPM. As a result, it is important to investigate the fracture behavior of the FGPM with a crack.

For FGPM, a series of crack problems have been studied in the past years. In general, in order to analytically solve the crack problems of FGPM, the material properties of FGPM are usually assumed to be an elementary function with respect to spatial coordinates, such as, linear function and exponential function [2-3]. So far, it is worth noting that very few certain assumed property distributions presented in the literature can be used for obtaining the analytical solutions of the crack problem in FGPM. Since certain assumed property distributions presented in the literature must be used with care, as they are not physically realizable for certain material combinations [4], it is necessary to systematically investigate fracture mechanics in FGPM with arbitrarily distributed properties. For FGPM with arbitrarily distributed properties, the governing equations of crack problem may become partial differential equations with variable coefficients. It is very difficult to obtain the analytical solutions of the governing differential equations. In order to overcome the complexity of the mathematics involved, a piecewise-exponential model [5-6] is proposed to investigate the arbitrarily oriented crack problem in FGPM with arbitrary properties in this paper.

2. Piecewise - exponential model

Consider an infinite FGPM strip that contains a Griffith crack n, as shown in Fig. 1. The thickness of the strip is h, the length of the crack are a-b, a and b denote x-coordinates of both crack-tips, respectively. Assume the material properties of FGPM vary along the x-axis, the constitutive and equilibrium equations can be written as

$$\begin{aligned} \tau_{xz} &= c_{44}\frac{\partial w}{\partial x} + e_{15}\frac{\partial \phi}{\partial x} \quad D_x = e_{15}\frac{\partial w_n}{\partial x} - \varepsilon_{11}\frac{\partial \phi}{\partial x} \\ \tau_{yz} &= c_{44}\frac{\partial w}{\partial y} + e_{15}\frac{\partial \phi}{\partial y} \quad D_y = e_{15}\frac{\partial w_n}{\partial y} - \varepsilon_{11}\frac{\partial \phi}{\partial y} \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \\ &\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0 \end{aligned} \tag{2}$$

The boundary conditions are：

$$\begin{cases} \tau_{xz}(0,y) = D_x(0,y) = 0 \\ \tau_{xz}(h,y) = D_x(h,y) = 0 \end{cases} \tag{3}$$

The shear modulus, piezoelectric coefficient and dielectric parameter of FGPM are assumed to vary in the x-direction according to the following expressions

$$[c_{44}, e_{15}, \varepsilon_{11}] = [c_{440}, e_{150}, \varepsilon_{110}]\, f(x) \tag{4}$$

As for the material parameters is an arbitrary function of x-coordinate, the governing equations may become partial differential equations with variable coefficients. In order to simulate the arbitrary variations of the shear modulus, piezoelectric coefficients and dielectric parameters, the piecewise exponential model can be employed. The FGPM strip is divided into *m* layers along thick, as shown in Fig. 2. Each sub-layer is marked by a subscript n ($n = 1,2\ldots\ldots m$). The thickness of the nth layer is $h_n - h_{n-1}$. In each sub-layer, the material parameters are assumed to be an exponential function, and they are continuous at sub-interfaces.

$$[c_{n44}, e_{n15}, \varepsilon_{n11}, \rho_n] = [c_{n440}, e_{n150}, \varepsilon_{n110}, \rho_{n0}]\, e^{\delta_n x} \tag{1}$$

The constitutive and equilibrium equations (1-2) can be reduced to

$$\begin{cases} \tau_{nxz} = c_{n44}(x)\dfrac{\partial w_n}{\partial x} + e_{n15}(x)\dfrac{\partial \phi_n}{\partial x} \\ \tau_{nyz} = c_{n44}\dfrac{\partial w_n}{\partial y} + e_{n15}\dfrac{\partial \phi_n}{\partial y} \end{cases} \tag{6}$$

$$\begin{cases} D_{nx} = e_{n15}\dfrac{\partial w_n}{\partial x} - \varepsilon_{n11}\dfrac{\partial \phi_n}{\partial x} \\ D_{ny} = e_{n15}\dfrac{\partial w_n}{\partial y} - \varepsilon_{n11}\dfrac{\partial \phi_n}{\partial y} \end{cases} \tag{7}$$

$$\begin{cases} \dfrac{\partial \tau_{nxz}}{\partial x} + \dfrac{\partial \tau_{nyz}}{\partial y} = 0 \\ \dfrac{\partial D_{nx}}{\partial x} + \dfrac{\partial D_{ny}}{\partial y} = 0 \end{cases} \tag{8}$$

Then, the boundary conditions (3) is reduced to:

$$\begin{cases} \tau_{1xz}(0,y) = D_{1x}(0,y) = 0 \\ \tau_{mxz}(h,y) = D_{mx}(h,y) = 0 \end{cases} \tag{9}$$

The continuity conditions can are written as：

$$\begin{cases} \tau_{nxz}\left(h_n,y\right) = \tau_{(n+1)xz}\left(h_n,y\right) \\ D_{nxz}\left(h_n,y\right) = D_{(n+1)xz}\left(h_n,y\right) \\ w_n\left(h_n,y\right) = w_{n+1}\left(h_n,y\right) \\ \phi_n\left(h_n,y\right) = \phi_{n+1}\left(h_n,y\right) \end{cases} \tag{10}$$

Based on the above analysis, the piecewise exponential model is used to investigate the arbitrarily oriented crack problem in FGPM with arbitrary properties.

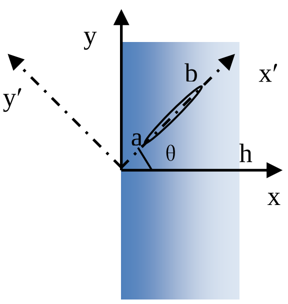

Fig.1 Geometry of FGPM with arbitrary properties

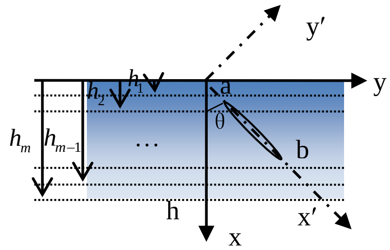

Fig.2 Piecewise-exponential model for FGPM with arbitrary distributed properties

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