Multi-objective Reliability-based Topology Optimization of Structures using Fuzzy Set Model

S. Sleesongsom[1*], S. Bureerat[2]

[1]Department of Aeronautical Engineering, International Academy of Aviation Industry, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand
[2] Sustainable and Infrastructure Development Center, Department of Mechanical Engineering, Faculty of Engineering, KhonKaen University, KhonKaen City 40002, Thailand

[*]Corresponding author: suwin.se@kmitl.ac.th

1. Introduction

Optimum topology has been developed in [1, 2] exactly is depended on material properties, external loading and other conditions. If uncertainties caused factors are vary that make the optimum design cannot used in practical or unreliable optimize result [3]. To address the problem there are two main strategies to account for uncertainties in topology optimization problem: robust topology optimization (RTO) [4] and reliability-based topology optimization (RBTO) [5]. The first technique is to optimize the expectation and variability of system performance with respect to uncertainties simultaneously, through which the robustness of system performance can be improved, while the second is concerned about failure probability constraints when optimizing the system performance, through which reliable optimization design can be achieved. Both methods are based on probabilistic model or non-probabilistic model [3]. The first model is the most popular due to its progress, but this technique requires precise on the statistical distribution of uncertainties. A good distribution of uncertainties is required a large amount of objective information, which is more computational time in practical preliminary design stage. In opposite with the second model, which is non-probabilistic models that the well-known of these techniques is fuzzy set method [6]. The fuzzy set model is an alternative technique due to it gives moderate conservative result. It is the best choice to collect the uncertainties in to RBTO by using level set to soft separate between the members and non-members of the set. It makes the model can get an acceptable solution. The disadvantage of the present RBTO is still complex in analysis due to the combination of the fuzzy set into the topology optimization problem is triple-loop nest problem includes the double loop nest in finding possibilistic safety index (PSI) and topology optimization. Later it has been solved using the target performance-based design approach as a result to the triple loop can reduce to double-loop nested problem [4]. The target performance-based approach is changed the PSI into the target performance of the ith constraint and called the equivalent possibilistic safety index (EPSI) by minimizing the constraint at some level cut. This technique can open to add the experience of the expert opinion to choose the level cut or membership level into RBTO.

Aim of this research to reduce the complicate of double-loop nest problem in RBTO using multi-objective optimization technique with fuzzy uncertainties.

2. Method

The possibility index can apply to multi-objective topology optimization problem, which incorporates with the fuzzy set method to deal with the uncertainties:

Min $\{f(\boldsymbol{\rho}), \pi_f^{max}\}$ (1)
Subject to
$\pi_f^{max} = \max((\mathrm{Pos}(g_i(\boldsymbol{\rho},\mathbf{a}) \le 0)) \le 1)$ $i = 1,2,..., N$
$\mathbf{0} < \boldsymbol{\rho}^l \le \boldsymbol{\rho} \le \boldsymbol{\rho}^u$

where π_f is possibilistic safety index**,** $\mathbf{a}$ is fuzzy variable vector and $g_i(\boldsymbol{\rho},\mathbf{a})$ become fuzzy rather than crisp. The new multi-objective reliability topology optimization problem is established in the possibility context.

For solving the multi-objective topology optimization problem in Eq. (1) is triple-loop nested problem, which is computational burden. The problem can reduce to double-loop problem by using the target performance-based approach [3] that has been proved the equivalent of the original failure possibility π_f^{max} in (1) and the new one:

Min $\{f(\boldsymbol{\rho}), EPSI\}$ (2)
Subject to
$EPSI = g_i(\boldsymbol{\rho},\mathbf{a}^{\pi_{fi}}) \ge 0$, $\pi_{fi} \in [0,1]$, $i = 1,2,..., N$
$\mathbf{0} < \boldsymbol{\rho}^l \le \boldsymbol{\rho} \le \boldsymbol{\rho}^u$

where $\mathrm{Min}(g_i(\boldsymbol{\rho},\mathbf{a}^{\pi_{fi}})$ is called the target performance of the constraint.

. Two design examples are used to demonstrate the present technique, which has the objectives are mass and compliance of the structure, while the second example is added constraint of structural strength. The multi-objective topology optimization problem can be formulated in MOP1 and MOP2 when it corporate with fuzzy set model.

$$\min_{\rho^{GEF}} \{c, r, EPSI\} \quad (3)$$

Subject to

$EPSI = g_i(\boldsymbol{\rho}, \mathbf{a}^{\pi fi}) \geq 0$, $\pi_{fi} \in [0,1]$, $i = 1,2,..., N$

$g_1 = c \leq 5c_{\min}$

$g_2 = 0.2 \leq r \leq 0.8$

$g_3 = \sigma_{\max}^{eqv} \leq \sigma_{yt}$ for MOP2

$\rho_i \in \{0.0001\text{m}, 1\text{m}\}$ for MOP1

$\rho_i \in \{0.000001\text{m}, 0.01\text{m}\}$ for MOP2

where ρ_i^{GEF} is the value of ith design variable, ρ_i is the thickness of ith finite element, m is the structural mass, $r = m(\boldsymbol{\rho})/m(\boldsymbol{\rho}^u)$ is the normalized mass or ratio of structural mass to maximum mass, c is the structural compliance, $c_{\min} = c(\boldsymbol{\rho}^u)$, $\sigma_{\max}^{eqv}$ is the maximum value of Von Mises stress of the ground element, and **a** is fuzzy variable (E, ν, and F). The last constraint in the design problem is bound constraints.

OMPBIL is used for solving the optimisation problem in this research due to its performance, which has been studied in [1]. The parameters of optimizer set accord with our previous study [1].

The topological design domain and loads are shown in Fig. 2. The uncertainties are Young's modulus E, Poisson's ratio v, and load F, which is assumed to be fuzzy variables. The membership function of the variables is triangular shaped with values are E = (190, 200, 210) ×10^9 N/m^2, v = (0.25, 0.3, 0.35), and F = (0.9, 1, 1.5) kN, while the tensile yield strength σ_{yt} = 200 × 10^6 N/m^2 is not considered to be a fuzzy variable.

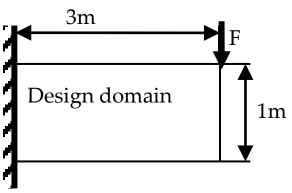


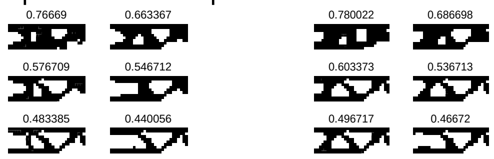
Fig. 2. Structural design domains

3. Results

Some selected optimal topology designs obtained from the proposed MORBTO are shown in Fig. 3(b) and 4(b). As a comparison, the optimal topology design obtained from the deterministic topology optimization is also presented in Fig 3(a) and 4(a). In the deterministic topology optimization, initial data are followed our previous work [2]. It can be seen from Fig. 3(b) and 4(b) that the MORBTO under the EPSI yields optimal topology design different from those yielded by the deterministic topology optimization (Fig. 3(a) and 4(a)).

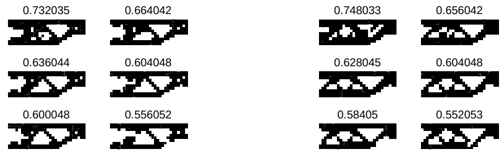
4. Conclusion

The proposed MORBTO technique can reduce complexity due to double-loop problem as the previous technique, which performed in single objective optimization problem. The present technique can accomplish it at the same time in one optimization run using multi-objective technique. Two design examples are used to demonstrate the present technique, which has the objectives are mass and compliance of the structure, while the second example is added constraint of structural strength. The results show the proposed technique is made the structure of the structural topology optimization is more conservative, and realizable, which simpler than the previous technique.



(a) (b)

Fig. 3. The optimal topology design of MOP1
(a) π_{fi} = 1 (b) π_{fi} = 0.001.



(b) (b)

Fig. 4. The optimal topology design of MOP2
(a) π_{fi} = 1, (b) π_{fi} = 0.001.

Acknowledgment

The authors are grateful for the financial support provided by *King Mongkut's Institute of Technology Ladkrabang*, the Thailand Research Fund (BRG5880014).

References

[1] S. Sleesongsom and S. Bureerat, Topology Optimisation Using MPBILs and Multi-Grid Ground Element, *Appl. Sci.*, 8 (2018) 271.

[2] D. Guirguis and M.F. Aly, A derivative-free level-set method for topology optimization. *Finite Elem. Anal. Des.*, 120 (2016) 41–56.

[3] H. Yin, D. Yu, and R. Xia, Reliability-based topology optimization for structures using fuzzy set model, *Comput.Methods Appl. Mech. Engrg*, 333 (2018) 197-217.

[4] P. Dunning and H. Kim, Robust topology optimization: minimization of expected and variance of compliance, *AIAA J*. 51 (11) (2013) 2656–2664.

[5] G. Kharmanda, N. Olhoff, A. Mohamed, and M. Lemaire, Reliability-based topology optimization, *Struct. Multidiscip. Optim.* 26 (2004) 295–307.

[6] B. Moller, W. Graf, and M. Beer, Fuzzy structural analysis using α-level optimization, *Comput. Mech*. 26 (2000) 547–565.